I.E.S./I.S.S. Examination => 2011

E-JTT-L-TUD

STATISTICS – IV

Time Allowed : Three Hours | *Maximum Marks : 200*

INSTRUCTIONS

Candidates should attempt FIVE questions in all including Question nos. 1 and 5 which are compulsory and attempt remaining THREE questions by choosing at least ONE each from Sections A and B.

The number of marks carried by each question is indicated at the end of the question.

Answers must be written in ENGLISH.

Symbols and abbreviations are as usual.

If any data is required to be assumed for answering a question, it may be suitably assumed, indicating this clearly.

SECTION A

1. Attempt any ***five*** parts :

(a) Define persistent, transient and ergodic states. Show that if $i \leftrightarrow j$ and if state i is persistent, then state j is also persistent. *8*

(b) For the linear growth process with the usual notations and $X(0) = m$, show that
$E\{X(t)\} = m \exp\{(\lambda-\mu)t\}$. *8*

(c) Prove that every basic feasible solution to a linear programming problem (L.P.P.) corresponds to an extreme point of the convex set of constraints of the L.P.P. and conversely. *8*

(d) Explain duality in linear programming. Write down the dual of the following linear programming problem :

Minimize $Z = 8x_1 - 5x_2 + 3x_3$

subject to $3x_1 + 5x_2 \geq 20$

$4x_1 - 5x_3 \leq 12$

$5x_1 + 9x_2 - 2x_3 = 25$

$x_1, x_2, x_3 \geq 0.$ *8*

(e) Describe a two period inventory problem when the demand is probabilistic. How do you solve it ? *8*

(f) Describe the linear congruential method and the rejection technique of generating pseudo random numbers. *8*

2. (a) Let $\{\xi_n, n \geq 1\}$ be a sequence of i.i.d.r.v.'s with

$P(\xi_k = i) = i/15, i = 1, 2, \ldots 5.$

Define $X_n = \max_{1 \leq i \leq n} \{\xi_i\}, n = 1, 2, \ldots$.

Show that $\{X_n, n \geq 1\}$ is a Markov chain. Obtain its one-step transition probability matrix. *10*

(b) Consider a random walk with reflecting barriers on the state space $S = \{1, 2, ..., N\}$ and the transition probability matrix given by

$$P = \begin{bmatrix} q & p & 0 & 0 & . \\ q & 0 & p & 0 & . \\ 0 & q & 0 & p & \\ \vdots & & & & \\ & & & q & p \end{bmatrix}$$

Classify the states of the chain and obtain its invariant distribution. *10*

(c) Show that for a Poisson process $\{X(t), t \geq 0\}$ the correlation coefficient between $X(t)$ and $X(t + s)$ is $\left\{\frac{t}{t+s}\right\}^{\frac{1}{2}}$. *10*

(d) Describe Galton-Watson branching process. For a Galton-Watson branching process $\{X_n, n = 0, 1, 2, ...\}$ with $E(X_1) = m$ show that $E\{X_{n+r} \mid X_n\} = X_n m^r$, for $r, n = 0, 1, 2, ...$. *10*

3\. (a) Given a basic feasible solution to a L.P.P., explain how you will derive a new improved basic feasible solution. *10*

(b) State and prove the fundamental theorem of duality. *10*

(c) Stating the assumptions, derive the (s, S) inventory policy and show that it is optimal. *10*

(d) Describe a method of simulating the $M \mid M \mid 1 \mid \infty$ queueing system and a method of estimating the mean queue length and mean waiting time in the queue based on the simulated data. *10*

4. (a) For a persistent non-null state j with period d, show that $\lim_{n \to \infty} p_{jj}^{(n)} = d/\mu_j$. *10*

(b) The offspring distribution for a branching process is given by $p_0 = 1/2$, $p_1 = 1/3$ and $p_2 = 1/6$. *10*

(i) Derive the distribution of X_2, the size of the third generation and hence obtain $E(X_2)$ and $Var(X_2)$.

(ii) Determine the probability of ultimate extinction.

(c) Solve the transportation problem with 3 origins and 4 destinations whose transportation costs are given in the following table : *10*

	Destination				
Origin	D_1	D_2	D_3	D_4	Availability
O_1	1	2	3	4	30
O_2	7	6	2	5	50
O_3	4	3	2	7	35
Requirement	15	30	25	45	

(d) The following failure rates have been observed for a certain type of electronic component :

Week	1	2	3	4	5	6	7	8
Probability of failure by the end of week	0·05	0·13	0·25	0·43	0·68	0·88	0·96	1·00

Initially there are 1000 components in use. The cost of replacing an individual component is ₹ 1·25. If all the components are replaced as group, it costs ₹ 0·30 per component. Examine whether group replacement is better than individual replacement. *10*

SECTION B

5. Attempt any ***five*** parts :

(a) What is a life table ? Explain a method of constructing life table. With usual notations, prove that $q_x = \frac{2m_x}{2 + m_x}$. 8

(b) Explain how population growth is measured. Describe the logistic model of population growth and explain its properties. 8

(c) What is meant by adjusted measures of mortality ? Explain the direct and indirect methods of adjusting. 8

(d) Explain with examples the following types of errors in programming : 8

(i) Syntax error

(ii) Data error

(iii) Logical error

(iv) Run-time error

(e) Explain the features and uses of (i) sequential file and (ii) random access file. Give examples. 8

(f) Explain the steps for the security of data in a database. 8

6. (a) What is an abridged life table ? Describe (i) Reed-Merrell method and (ii) King's method of construction of abridged life table. 10

(b) Define General Fertility Rate (GFR) and Age Specific Fertility Rate (ASFR). Describe the methods for computing these fertility rates. Indicate why ASFR is considered as an improvement over GFR. 10

(c) Explain Makeham's Law of Mortality. How do you fit this for the given data ? 10

(d) What is meant by stable population ? Explain Lotka's model for stable population analysis. 10

7. (a) Explain the principles of design and analysis of algorithms. Illustrate with an example. 8

(b) Explain the different topologies used in a computer network. Distinguish between LAN, MAN and WAN. 8

(c) Explain (i) database, (ii) data warehouse, (iii) data mining. 8

(d) Draw a flow chart to compute the different components of a life table. 8

(e) Explain the process of data transmission and the process of error control. Explain any one error control coding technique. 8

8. (a) Define force of mortality μ_x and complete expectation of life $\overset{o}{e}_x$ and derive a relationship between them. 8

(b) Compute (i) general fertility rate, (ii) total fertility rate and (iii) gross reproduction rate from the data given below. Assume that for every 100 girls, 106 boys are born. 8

Age-group of child bearing females	Number of women	Age-specific fertility rate (per 1000)
15 – 19	212,019	98·0
20 – 24	198,732	169·6
25 – 29	162,800	158·2
30 – 34	145,362	39·7
35 – 39	128,109	98·6
40 – 44	106,211	42·8
45 – 49	86,753	16·9

(c) Explain (i) expert system and (ii) decision support system with suitable examples. 8

(d) Examine the concept of internet security, the relevant threats and their solutions. 8

(e) Explain (i) internal migration (ii) international migration and (iii) net migration. How are these migration rates computed ? 8